A PROPOSED METHOD FOR SHIP ROUTING USING LONG RANGE WEATHER FORECASTS

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ABSTRACT

Two advances in the calculus of variations method for minimal time ship routing are described. The first is a scheme for constructing ocean wave field forecasts which may be expected to have considerable skill for perhaps a week. The second is an improved technique for varying time extremal ship tracks toward admissibility. Both ideas are illustrated by calculating the optimum track ship route of a VC2AP3 vessel on a transpacific voyage. Possible future developments are discussed.

1. INTRODUCTION

The use of calculus of variations methods in computing minimal time ship routes has been restricted severely in the past by the unavailability of ocean wave field forecasts for extended periods. In an intitial attempt to remedy this situation it is proposed here that the wave forecasts now available for periods up to 2 days from the Fleet Numerical Weather Facility may be extrapolated with considerable skill for 5 more days by using certain 5- and 30-day forecasts now available from the ESSA Weather Bureau. Bleick and Faulkner [1] gave a method of computing minimal time ship routes by varying time extremal ship tracks toward the admissibility of reaching a desired terminal point. Their scheme of extremal variation is refined here so that rapid convergence of the track iteration process is assured. An example of wave field construction and extremal variation is given for a Pacific voyage.

2. WAVE FIELD CONSTRUCTION

The scheme of incorporating weather forecasts into the construction of a 10-member computer-stored time series of wave fields for the numerical example of a transpacific voyage consists of the following parts:

a) The Fleet Numerical Weather Facility prepares wave analyses at 00 and 12 GMT each day, as well as operational wave predictions at 12-hr. intervals for periods up to 48 hr. The first five members of the time series consisted of the analysis at 12 GMT of July 26, 1966, and the predictions for 00 and 12 GMT of July 27 and 28. The wave forecasts are based on winds generated from sea level pressure prognoses with consideration given to surface friction and static stability.

b) The ESSA Weather Bureau's 5-day surface pressure forecast, issued every Monday, Wednesday, and Friday, consisting of one sea level pressure map per day at 1230 GMT, was used to construct the next three members of the time series. The last three maps of the forecast issued on Wednesday, July 27, 1966, were used to determine surface winds and, in turn, to calculate height, period, and direction of the wind waves and swell. These data were used for the time series members at 12 GMT of July 29, 30, and 31.

c) The ESSA Weather Bureau's 30-day forecast was utilized for calculating the 9th and 10th members of the time series.

The 30-day forecasts issued to the public consist only of qualitative precipitation and temperature anomalies for continental United States and hence are not suitable for making long range wind estimates. However, the Weather Bureau's Extended Forecast Division does prepare 30-day forecasts of the mean surface pressure for the Northern Hemisphere in connection with their long range precipitation and temperature forecasts. Although not published for use outside the Weather Bureau, these pressure prognoses were made available to the authors in order to make a preliminary estimate of their possible usefulness in optimum ship routing. Surface winds were therefore estimated from this single map and again wave conditions were determined. The calculations were repeated on a daily basis using the same mean pressure map. Since the same winds are used over and over again, the predicted wave fields reach a steady state within a few days. The limited amount of computer core storage made it necessary to terminate the time series by using the above data for 12 GMT on August 1 and 2, with the last member of the time series being used to satisfy any later need for wave fields.

It is recognized that the predicted 30-day mean pressure map will invariably have relatively weak pressure gradients, as would be expected from the implied averaging process. In contrast, the individual daily charts which would make up such a mean would normally have considerably stronger gradients and thus strong winds and high seas, particularly in the vicinity of the migratory cyclones. Notwithstanding, the hypothesis was made that these prognostic mean charts may serve as an indicator of the principal storm tracks, which is really the primary consideration in routing ships for long voyages. As a preliminary test of this hypothesis, Lt. D. M. Truax constructed wave fields based on the 5- and 30-day forecasts and also from the observed climatological mean maps. Both were compared to the hemispheric wave fields analyzed twice-daily at the Fleet Numerical Weather Facility. The root mean square error for waves derived from the 5- and 30-day forecasts was only half the error resulting from the use of the climatological mean map (3 ft. vs. 6 ft.). While the sample size was not large enough to be conclusive, the result gave considerable encouragement that the long range forecasts had sufficient skill to serve as a useful guide for optimum ships routing. Nevertheless it is desirable to consider other ways of providing wave estimates for the latter part of a voyage extending beyond 5 days. Some suggestions for other possible methods are discussed later.

3. VARIATION OF EXTREMALS

As in the previous work [1] let the differential equations of a ships motion in the stereographic plane be

$$\dot{x}=V\cos p,\ \dot{y}=V\sin p. \tag{1}$$

It was shown that the ship track direction angle p on a time extremal route is

$$p=\arctan(\mu/\lambda)+\arctan(V_p/V). \tag{2}$$

Here

$$\lambda=\lambda_1\cos\alpha+\lambda_2\sin\alpha \tag{3}$$

and

$$\mu=\mu_1\cos\alpha+\mu_2\sin\alpha \tag{4}$$

are linear combinations of the linearly independent solutions λ_1,μ_1 and λ_2,μ_2 of the system of equations

$$\dot{\lambda}+V_x(\lambda\cos p+\mu\sin p)=0 \tag{5}$$

$$\dot{\mu}+V_y(\lambda\cos p+\mu\sin p)=0, \tag{6}$$

where the initial values $\lambda_1(0)=\mu_2(0)=1$ and $\mu_1(0)=\lambda_2(0)=0$ are used. If the arctan (V_p/V) term in (2) is ignored, it is seen that α is approximately the departure angle between the ship track and the Ox coordinate axis at the $t=0$ initial point of the voyage as indicated in figure 1. In the previous work [1] on varying a time extremal ship track toward the admissibility of reaching a desired terminal point the variation δp was considered to be dependent on the variation $\delta\alpha$ only. This is a convenient approximation to avoid mathematical complications, but its use may lead to a marked slowing down of the Newton-Raphson track iteration process. If this approximation is abandoned in computing the variations $\delta x,\delta y$ of a time extremal ship track solution of (1) and (2), then the dependence of δp on all of the variations δx, δy, $\delta\lambda_1$, $\delta\lambda_2$, $\delta\mu_1$, $\delta\mu_2$ and $\delta\alpha$ must be considered. This complete variation δp is found from (2) to be

$$\delta p=[W_x\delta x+W_y\delta y+S^2(\lambda\delta\zeta-\mu\delta\xi+E\delta\alpha)]/D \tag{7}$$

where

$$W=V_p/V, \tag{8}$$

$$\delta\xi=\delta\lambda_1\cos\alpha+\delta\lambda_2\sin\alpha, \tag{9}$$

$$\delta\zeta=\delta\mu_1\cos\alpha+\delta\mu_2\sin\alpha, \tag{10}$$

$$S^2=(1+W^2)/(\lambda^2+\mu^2), \tag{11}$$

$$D=1+W^2-W_p, \tag{12}$$

$$E=\lambda_1\mu_2-\lambda_2\mu_1. \tag{13}$$

The variation of (1), the time differentiation of (9) and (10), and use of (5), (6), and (7) give the following nonhomogeneous system of equations to solve for the desired variations $\delta x,\delta y$:

$$\begin{aligned}\delta\dot{x}=(V_x\cos p-\mu QW_x)\delta x+(V_y\cos p-\mu QW_y)\delta y\\ +\mu S^2Q(\mu\delta\xi-\lambda\delta\zeta-E\delta\alpha),\end{aligned} \tag{14}$$

$$\begin{aligned}\delta\dot{y}=(V_x\sin p+\lambda QW_x)\delta x+(V_y\sin p+\lambda QW_y)\delta y\\ -\lambda S^2Q(\mu\delta\xi-\lambda\delta\zeta-E\delta\alpha),\end{aligned} \tag{15}$$

$$\begin{aligned}-\delta\dot{\xi}=S^{-1}(V_{xx}+VD^{-1}W_x^2)\delta x+S^{-1}(V_{xy}+VD^{-1}W_xW_y)\delta y\\ +(V_x\cos p-\mu QW_x)\delta\xi+(V_x\sin p+\lambda QW_x)\delta\zeta\\ +QEW_x\delta\alpha,\end{aligned} \tag{16}$$

$$\begin{aligned}-\delta\dot{\zeta}=S^{-1}(V_{yx}+VD^{-1}W_yW_x)\delta x+S^{-1}(V_{yy}+VD^{-1}W_y^2)\delta y\\ +(V_y\cos p-\mu QW_y)\delta\xi+(V_y\sin p+\lambda QW_y)\delta\zeta\\ +QEW_y\delta\alpha,\end{aligned} \tag{17}$$

where $Q=VS/D$. Equations (14) to (17) are integrated, with zero initial values at the $t=0$ initial point of the track and with $\delta\alpha=1$, to obtain the variations $\delta x(T)$ and $\delta y(T)$ at the $t=T$ terminal point. These variations are really the partial derivatives $\partial x(T)/\partial\alpha$ and $\partial y(T)/\partial\alpha$ since we have taken $\delta\alpha=1$. The Newton-Raphson equations for determining ΔT and $\delta\alpha$ on a varied time extremal track, which attempt to reduce the terminal errors $\Delta x(T)$ and $\Delta y(T)$ of the previous extremal track, are then

$$\begin{aligned}\dot{x}(T)\Delta T+[\partial x(T)/\partial\alpha]\delta\alpha=\Delta x(T)\\ \dot{y}(T)\Delta T+[\partial y(T)/\partial\alpha]\delta\alpha=\Delta y(T).\end{aligned} \tag{18}$$

In the numerical integration of (1), (5), (6), and (14) to (17) it is desirable to use a wave field interpolation formula which will guarantee the continuity of all terms of these equations where any of x, y, t assume grid values. A method which achieves this when interpolating in the time

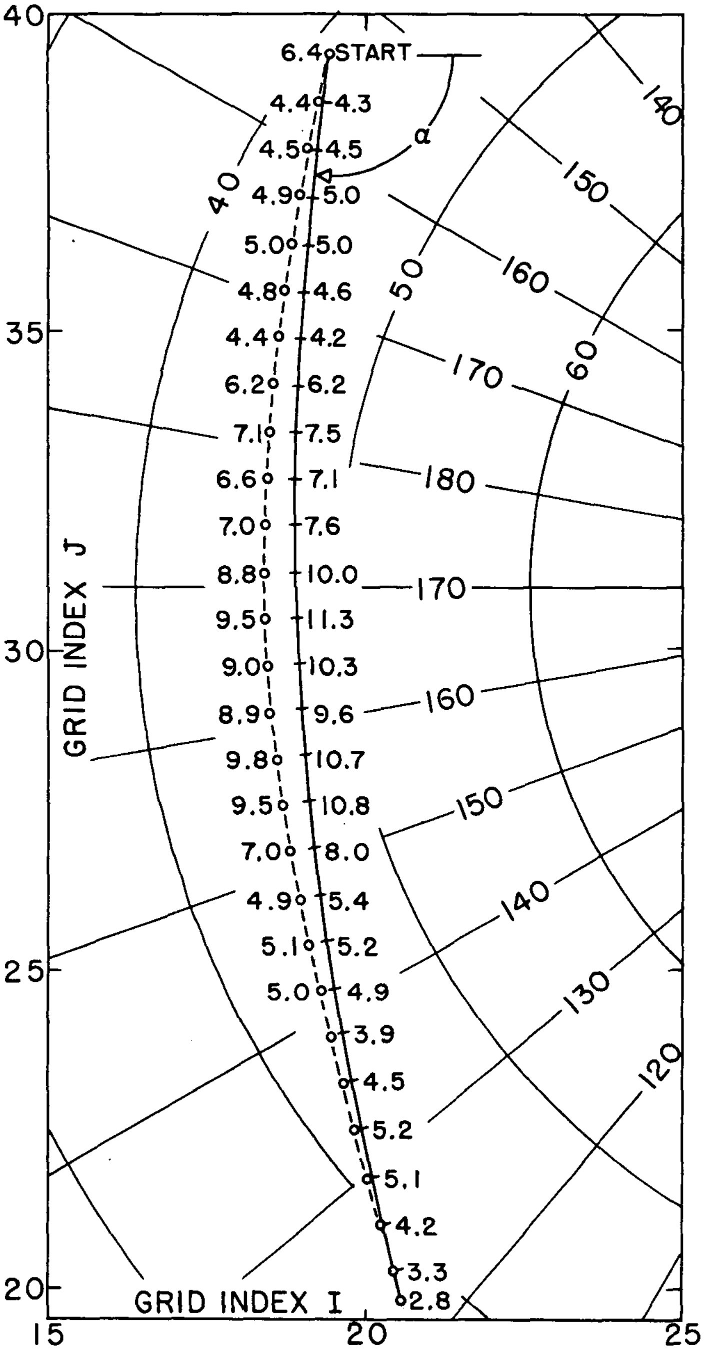

FIGURE 1.—Transpacific voyages of a VC2AP3. Wave heights in feet every 8 hr. on geodesic track (solid line) and minimal time track (circles).

dimension was given in [1]. The method given there for interpolating in the grid of the Oxy stereographic plane will not give the desired continuity of V_{xx}, V_{xy}, and V_{yy} of (16) and (17). The 16-point interpolation formula used here to assure the continuity of V and all its first and second order partial derivatives with respect to x and y is obtained from the 4×4 matrix $\mathbf{F}$, whose four rows and columns of function entries correspond to four successive x and y grid values, respectively. The interpolation mesh cell is the central cell of the array, with x and y measured from the cell center, and with the mesh distance considered to be two units. The formula is

$$F(x,y)=\mathbf{P}(x)\ \mathbf{F}\ \mathbf{P}'(y)/1024 \tag{19}$$

where the row matrix $\mathbf{P}(x)=[P_1,\ P_2,\ P_3,\ P_4]$ has the elements

$$\begin{aligned} P_1&=(x^2-1)(x-1)^2(x+2),\\ P_2&=(1-x)(3x^4+3x^3-9x^2-7x+18),\\ P_3&=(x+1)(3x^4-3x^3-9x^2+7x+18),\\ P_4&=(x^2-1)(x+1)^2(2-x), \end{aligned} \tag{20}$$

and the prime indicates matrix transposition. In contrast with the earlier work [1] it was found desirable to evaluate the various derivatives of $V=mv$ by explicit differentiation of the solution of the quadratic equation in v for the elliptical polar velocity diagram.

4. NUMERICAL EXAMPLE

Figure 1 illustrates the result of the new methods of wave field construction and time extremal track variation in the case of a transpacific voyage of a VC2AP3 vessel. The elliptical polar velocity diagram used was based upon the work of James [2]. The minimal time track starts from 154°E., 41°N., at 1200 GMT on July 26, 1966, and ends at 123°W., 38°N., at 0828 GMT on August 4, with circles indicating successive positions of the vessel at 8-hr. intervals. The solid line is the great circle route obtained by integrating (1) using

$$\begin{aligned} \cos p&=n[(31-J)/31.205]+m\\ \sin p&=n[(I-31)/31.205]-l \end{aligned} \tag{21}$$

where I, J are the Fleet Numerical Weather Facility stereographic plane grid indices, and l, m, n are the direction cosines of the normal vector to the great circle plane with respect to the $Oxyz$ axes. These cosines are computed from the normalized cross product of the vectors from the earth's center to the initial and terminal points of the track. No significant time difference between the geodesic and minimal time routes was found, but the latter did show a reduction in wave heights encountered as indicated in figure 1. The example illustrates the advantage of the new method of extremal variation in that the Newton-Raphson equations (18) were used as they stand without convergence difficulties, i.e., without resorting to the delayed approach to the limit scheme of using only some fraction of $\delta\alpha$ on the next track iteration. The new method also permitted the use of a rather large 4-hr. time step in the numerical integrations, with consequent gain in the speed of the iteration process. The Fortran computer program may be obtained from the authors.

5. CONCLUDING REMARKS

The example used in the illustration above together with Truax's comparison of calculated and observed wave heights provides some encouraging evidence that the extended range forecasts may furnish useful information for optimum ship routing. However, as mentioned earlier, it would be desirable to make more thorough tests and consider other sources of wave fields for the latter part of extended voyages, particularly beyond 5 days. As an alternative to using monthly climatological mean pressure maps to estimate surface waves, observed mean monthly wave heights and directions, as a function of latitude and longitude, could be utilized after the 5-day forecasts for the remainder of the trip; for example, see [3].

A further refinement in the development of a wave climatology would be a separation according to weather types that delineate the principal storm tracks, which may vary from week to week as well as by season. Application of such information would require a forecast of the weather type for the period of the voyage. Some efforts in this direction are being taken at the Fleet Numerical Weather Facility.

Of course, what is really wanted here is a once or twice daily individual sea level pressure prognosis for an entire transoceanic voyage which may be several weeks long. A number of groups are experimenting with long-range weather prediction by numerical integration of the hydrodynamical equations. It is expected that eventually such predictions may show skill for perhaps several weeks.

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